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New Approach to Solve Assignment Problems with Branch and Bound Algorithm

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Abstract: This paper presents a new branch and bound algorithm for solving the assignment problems that uses a lower bound based with branching procedure. This is special type of linear programming problem dealing with the allocation of various resources to the various activities on one to one basis. Most of the business organization operates in traditional and old fashioned manner. Decision making is based on judgment and is quite subjective. Consequently, the entire process of decision making may lead to uncertain and unwanted results. Industries need more scientific and data driven strategies to achieve higher goals and to avoid risks of failure. The assignment problem deals with the maximum profit assigning of jobs or objects to agents such that each job is assigned to precisely one person subject to capacity restrictions on the agents. A new algorithm for the generalized assignment problem is presented that employs both column and row generation and also branch and bound to obtain optimal integer solutions to a set partitioning formulation of the problem. First, a generalized assignment issue that is solved by an existing Hungarian approach that only employs column generation is solved in this study by employing both row and column generation, and the optimal solution is obtained, which is similarly similar. The study is classified according to the objective, solution methodologies and related considerations. This paper hopes to give the reader an idea about the best job assigning that is modeled in this paper, discuss the formulation of assignment problem, its efficacy, enumerate the benefits gained and identify areas for further improvement.

Keywords: Combinatorial Optimization, Assignment Problem, Lower Bounds, Branch and Bound, Row-Column Generation

1. Introduction

In a wide range of fields problems related to assignment arise, as like, healthcare, transportation, education, job assigning and sports. This is a well-studied topic in combinatorial optimization problems under optimization or operations research branches. Assignment problem has been used to solve many problems worldwide [1]. This problem has been commonly envisaged in many educational functions all over the world.

Assignment problem refers to the analysis on how to assign objects to objects in the best possible way [2, 3]. The assignment intend underlying combinatorial structure, while the objective function reflects the desires to be optimized as much as possible. Several diverse methods have been proposed [1, 2], such as the heuristics method [4], the exact method [5], the local search-based [6], the population search-based [7], and the hybrid algorithm [8]. The Assignment Problem can be solved using the linear programming technique introduced by Votaw and Orden [10], the transportation algorithm or the Hungarian method developed by Kuhn [10]. The Hungarian method is acknowledged to be the first applied method for solving the standard assignment problem. Balinski and Gomory [11] introduced a labeling algorithm for solving the transportation and assignment problems. An algorithm for solving fuzzy assignment problems that uses Robust ranking technique with fixed fuzzy numbers was introduced by Nagarajan and Solairaju [13]. Geetha et al. [14] first formulated cost time assignment problem has the multi criterion problem. In existing literature, several researchers developed different metrologies for solving generalized assignment problem. Among this, one may refer to the works of Ross et al. [15]. Bai et al. [16] proposed a method for solving fuzzy generalized assignment problem. However we will solve the assignment problem by the help of branch and bound techniques.

The aim of assignment problem is to discover an assignment among two or more sets of elements, which could minimize the total cost of all matched pairs and obviously using branching procedure. Hence, every assignment problem has a table or matrix form. Here, denotes objects or people to assign, while the columns the rows denotes the things or tasks to be assigned. The numbers in the table denote costs related to every particular assignment. With that, this study presents a review of assignment problem within educational activities, where the problems were classified into allocation problems and timetabling.

2. Definitions

2.1. Combinatorial Optimization

In mathematics, the term combinatorial is used to describe a type of problem which deals with problems with minimizing and maximizing a function of variables subject to inequality or equality constraints and integrality restrictions on all or some of the restrictions [9].

2.2. Assignment Problem

A basic combinatorial optimization problem is the assignment problem. This is special types of linear programming problem which deals with the allocation of various resources to various objects where the cost is minimized.

2.3. Lower Bound

Lower bound is the minimum cost of assigning jobs in assignment problems. The final or optimal result cannot be minimum than the lower bound.

2.4. Branch and Bound

Branch and bound method is a method based on enumeration of the possible solution of a combinatorial optimization problem. It is used to find optimal solution for combinatory, discrete and general mathematics. It uses the concept of trees, logic trees, bounds to solve optimization problem.

3. Assignment Problem Using the Branch and Bound Technique

The Assignment Problem of $n \times n$ cost matrix of real numbers is as follows.

As a result, there is a mathematical form of the generic assignment problem.

Minimize

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}\, x_{ij}$$

Subject to

$$\sum_{i=1}^{n} x_i = 1$$

$$\sum_{j=1}^{n} x_j = 1$$

$$x_{ij} = \begin{cases} 1 \text{ if jth job is assigned to the ith person} \\ \qquad 0 \text{ otherwise} \end{cases}$$

x_{ij} The decision variable, denotes the assignment of jth job to the ith object and a_{ij} denotes the cost of assigning. The Assignment problem has the general form where the objective is to assigning jth job to ith person exactly one job to one person and also the company should have its minimum cost for assigning jobs to earn more profits.

Table 1. *Cost matrix of job assignment problem.*

	$Job1$	$Job2$	$Job3$	$Job4$	$Jobj$	$JobN$
$Person\ 1$	a_{11}	a_{12}	a_{13}	a_{14}	$\dots a_{1j}$	a_{1n}
$Person\ 2$	a_{21}	a_{22}	a_{23}	a_{24}	$\dots a_{2j}$	a_{2n}
$Person\ 3$	a_{31}	a_{32}	a_{33}	a_{34}	$\dots a_{3j}$	a_{3n}
.	.	.	.	.	.	.
.	.	.	.	.	.	.
$Person\ i$	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{ij}	a_{in}
.	.	.	.	.	.	.
.	.	.	.	.	.	.
$Person\ N$	a_{n1}	a_{n2}	a_{n3}	a_{n4}	$\dots a_{n4} \dots$	a_{nn}

3.1. Algorithm

1. Firstly determine the lowest bound for assigning the jobs by choosing row minimum and summing over. This is the minimum cost for the problem.
2. Then assign a job to one person or machine randomly.
3. After Leaving the respective row and column which job have been chosen randomly we have $(n-1)$ rows and $(n-1)$ columns. So $(n-1) * (n-1)$ matrix formed where choose the very minimum cost from this matrix form and also leaving this respective row and column.
4. Repeating this procedure and summing over get the cost for respective job for assigning.

3.2. Branching Guidelines

1. Assigning one job to one person or machine we have done one branching from this branching tree minimum cost have to be considered.
2. If there is a tie on the lower bound then the terminal node at the lower most is to be considered for the further branching.
3. The optimality is obtained if the lowest lower bound happiness is found at any of the terminal nodes at the (n-1)th level [16]. The assignments on the node's path to that node, as well as the missing pair of row-column combinations from the optimum solution are then added.

Numerical Illustrations:

Example 1

A company has four jobs to be done on four machine; any job can be done on any machine. The time in hour taken by the machines for the different jobs are as given below. Assign the machines to jobs so as to minimize the total machine hours.

10	12	9	11
5	10	7	8
12	14	13	11
8	15	11	9

Figure 1. *4×4 Cost matrix for job assignment problem.*

Firstly we have to check lowest bound for the job assigning problem. It's the minimum cost. We will do this by choosing one person for one job and then we will remove the corresponding row and column from consideration. As like if we assign job1 to candidate 1 then we will remove first row and first column from consideration and choose another person for another job from rest of the matrix form which is minimum from the rest of matrix form. Then find out the minimum cost from the matrix form and again remove this corresponding row and column and continue this procedure.

$$P_{11}^{1} = 10 + 7 + 9 + 14 = 40$$

$$P_{12}^{1} = 12 + 5 + 9 + 13 = 39$$

$$P_{13}^{1} = 9 + 5 + 9 + 14 = 37$$

$$P_{14}^{1} = 11 + 5 + 11 + 14 = 41$$

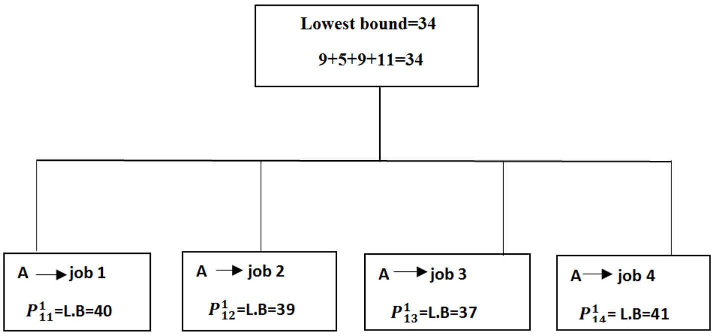

Figure 2. *Lower bound cost for assigning job to candidate A.*

From this box, the $P_{11}^{1}, P_{12}^{1}, P_{13}^{1}, P_{14}^{1}$ are the terminal nodes, from these nodes the lower most to be considered, the lowest bound for candidate A is 37. So A should be assigned to job3 so further branching will be start from this point. A is fixed for job3. Next check B for job1 or job2 or job4 as job3 has been assigned to job3. When we check for B; A to job3, so job3 corresponding row and column would be negligible from consideration and so for B. After doing first branching these following job left for candidates B, C and D.

10	12	9	11
5	10	7	8
12	14	13	11
8	15	11	9

Figure 3. *Assigning job3 to candidate A.*

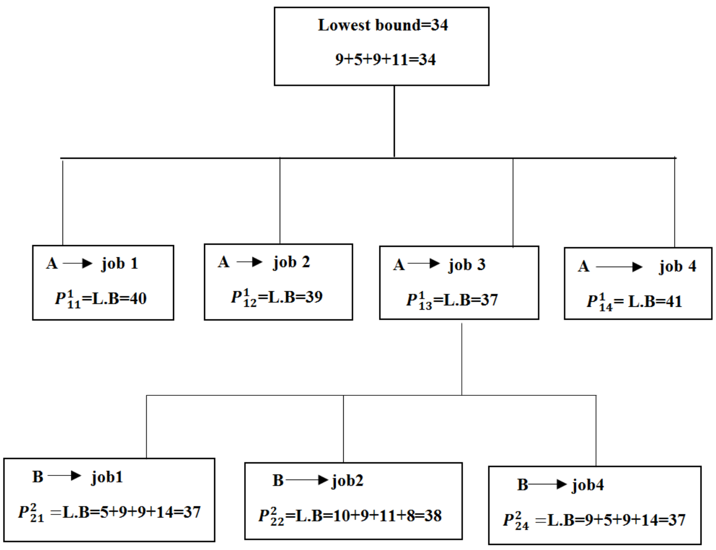

Figure 4. *Lower bound cost for assigning job to candidate B.*

Here, P_{13}^1, P_{21}^2, P_{22}^2, P_{24}^2 are terminal nodes but for P_{21}^2 and P_{24}^2 we have a tie here we have a tie. Then these terminal nodes at lower most to be considered. So let's choose P_{21}^2 for candidate B. Then further branching is needed for candidate C and D. After 2 branching we have these following job left for C and D.

If we assign A to job3, B to job1, C to job2, D to job4 would be the job assigning for the problem for minimum cost, Minimum cost is 37. Solving it, we get the solution as with the optimal objective value 37 which represents the optimal total cost. Which is the solution of the problem using proposed branch and bound techniques.

10	12	9	11
5	10	7	8
12	14	13	11
8	15	11	9

Figure 5. *Assigning job1 to candidate B.*

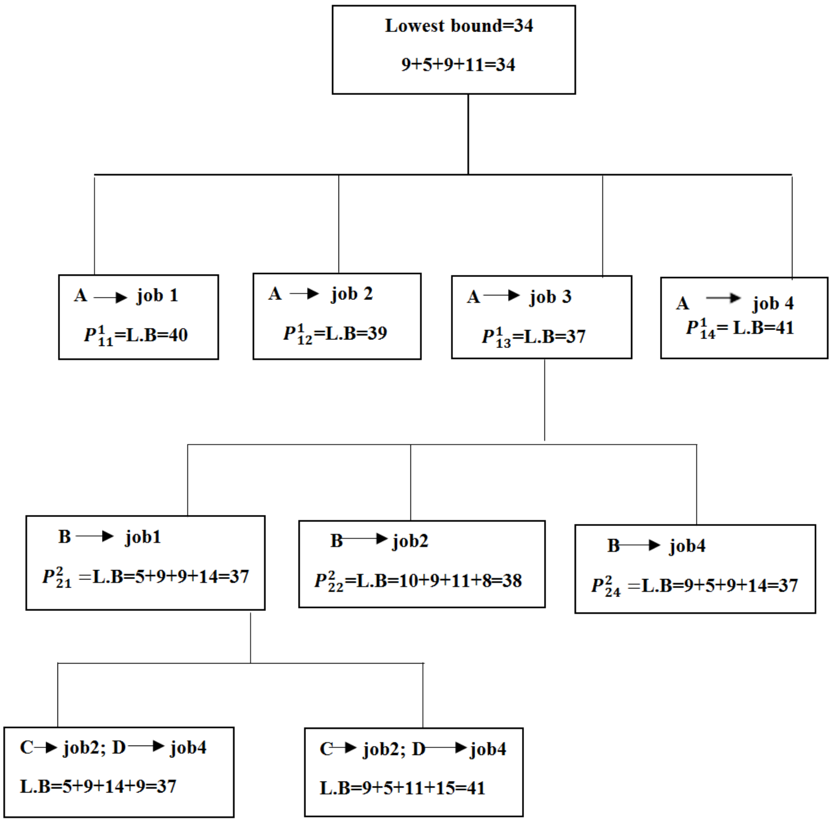

Figure 6. *Lower bound cost for assigning jobs to candidate C and D.*

Another Example:

$$P_{11}^1 = 2 + 1 + 2 + 3 + 9 = 17$$

$$P_{12}^1 = 9 + 1 + 3 + 4 + 9 = 26$$

$$P_{13}^1 = 2 + 1 + 2 + 3 + 5 = 13$$

$$P_{14}^1 = 7 + 1 + 2 + 4 + 9 = 23$$

$$P_{15}^1 = 1 + 2 + 3 + 5 + 7 = 18$$

Here P_{11}^1, P_{12}^1, P_{13}^1, P_{14}^1, P_{15}^1 are terminal nodes where P_{13}^1 is of minimum cost. So agent A will be assigned to job 3. For this reason further branching will be start from here. So we will remove corresponding row and column from the given data. After removing first row and 3rd column the matrix form will look like the following form. Further branching will start from here which will support minimum cost as like this manner.

2	9	2	7	1
6	8	7	6	1
4	6	5	3	1
4	2	7	3	1
5	3	9	5	1

Figure 7. *5×5 Cost matrix for job assignment problem.*

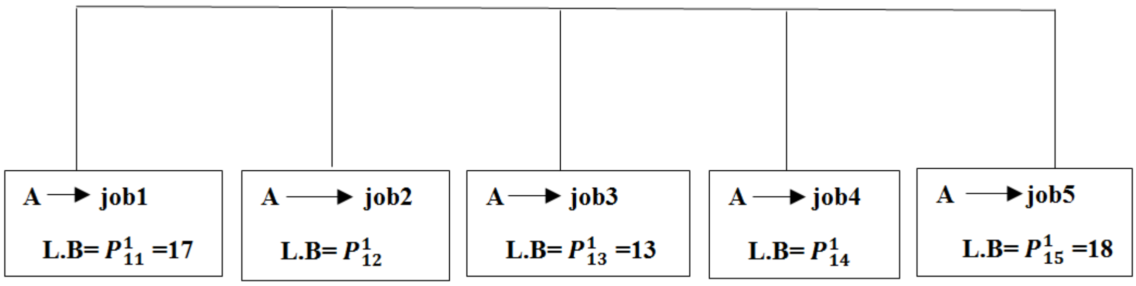

Figure 8. *Lower bound cost for assigning job to candidate A.*

2	9	2	7	1
6	8	7	6	1
4	6	5	3	1
4	2	7	3	1
5	3	9	5	1

Figure 9. *Assigning job3 to candidate A.*

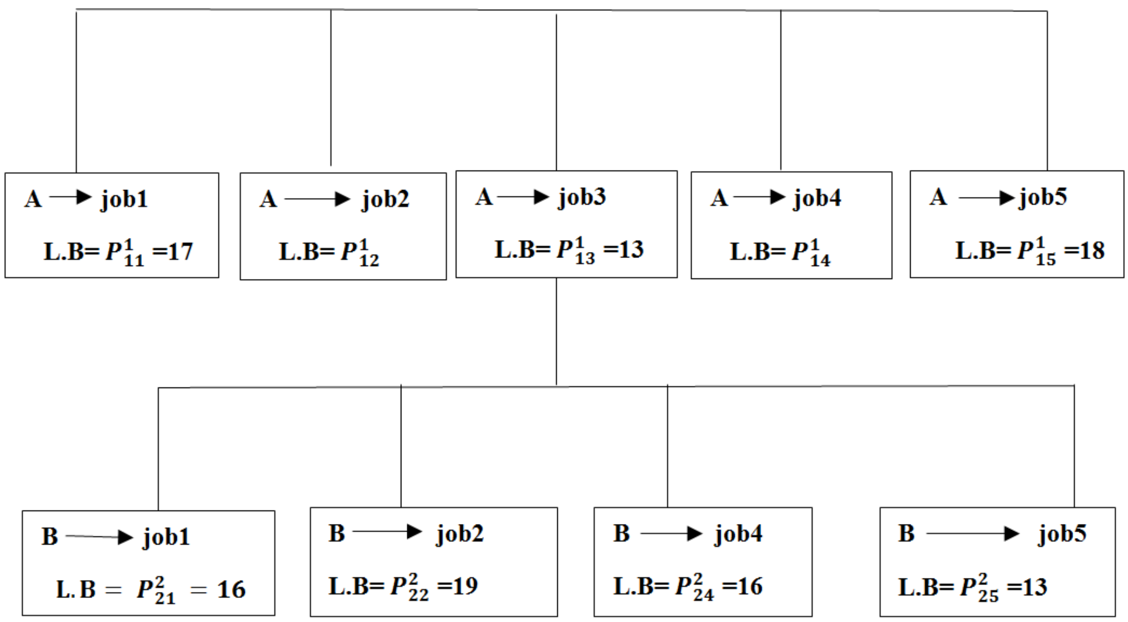

Figure 10. *Lower bound cost for candidate B.*

If we confirm A to Job3 because this is the lowest cost to assign among all others then further branching will start from there. Confirming candidate A to Job3 then, P^2_{21}, P^2_{22}, P^2_{24}, P^2_{25} are terminal nodes where we can choose our assignment. From this table we see that the lower most cost for B is 13 in P^2_{25} so B will assign to job5. So we can confirm candidate to job5. So by using algorithm we will remove the corresponding row and column from further consideration. After removing 2nd row and 5th column the matrix form will look like the following form. Further branching will start from there and same procedure will apply for the remaining matrix form which now we have. After removing second row and fifth column the matrix form will look like this,

2	9	2	7	1
6	8	7	6	1
4	6	5	3	1
4	2	7	3	1
5	3	9	5	1

Figure 11. *Assigning job5 to candidate B.*

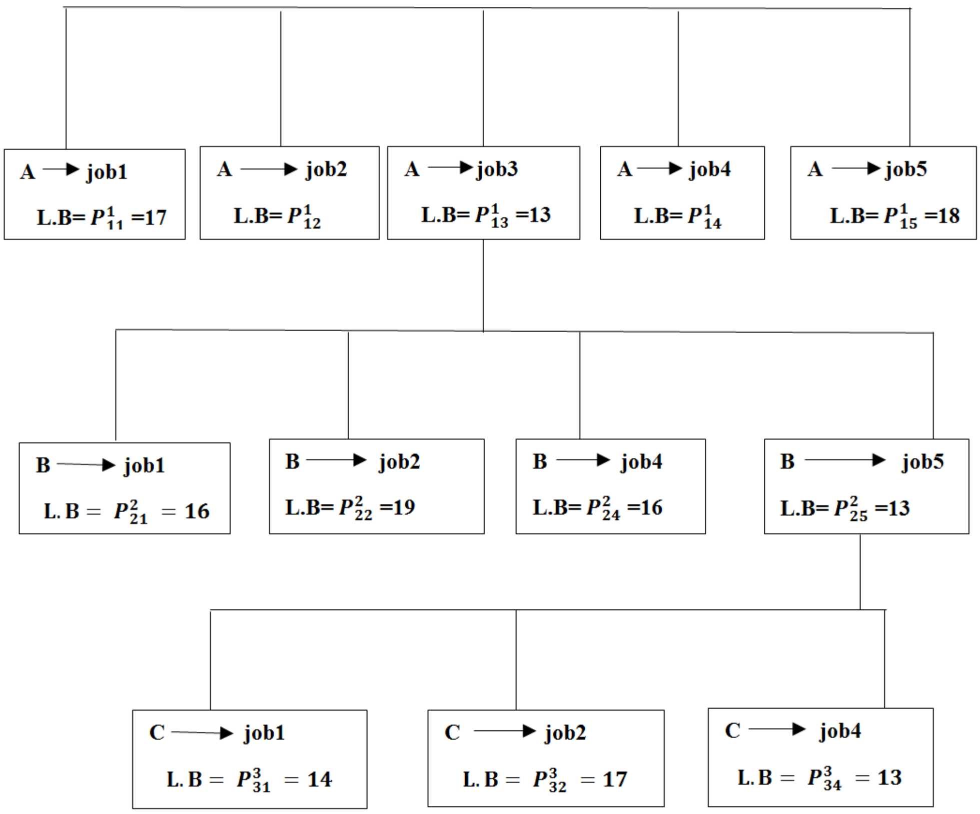

Figure 12. *Lower bound cost for assigning job4 to candidate C.*

2	9	2	7	1
6	8	7	6	1
4	6	5	3	1
4	2	7	3	1
5	3	9	5	1

Figure 13. *Assigning job4 to candidate C.*

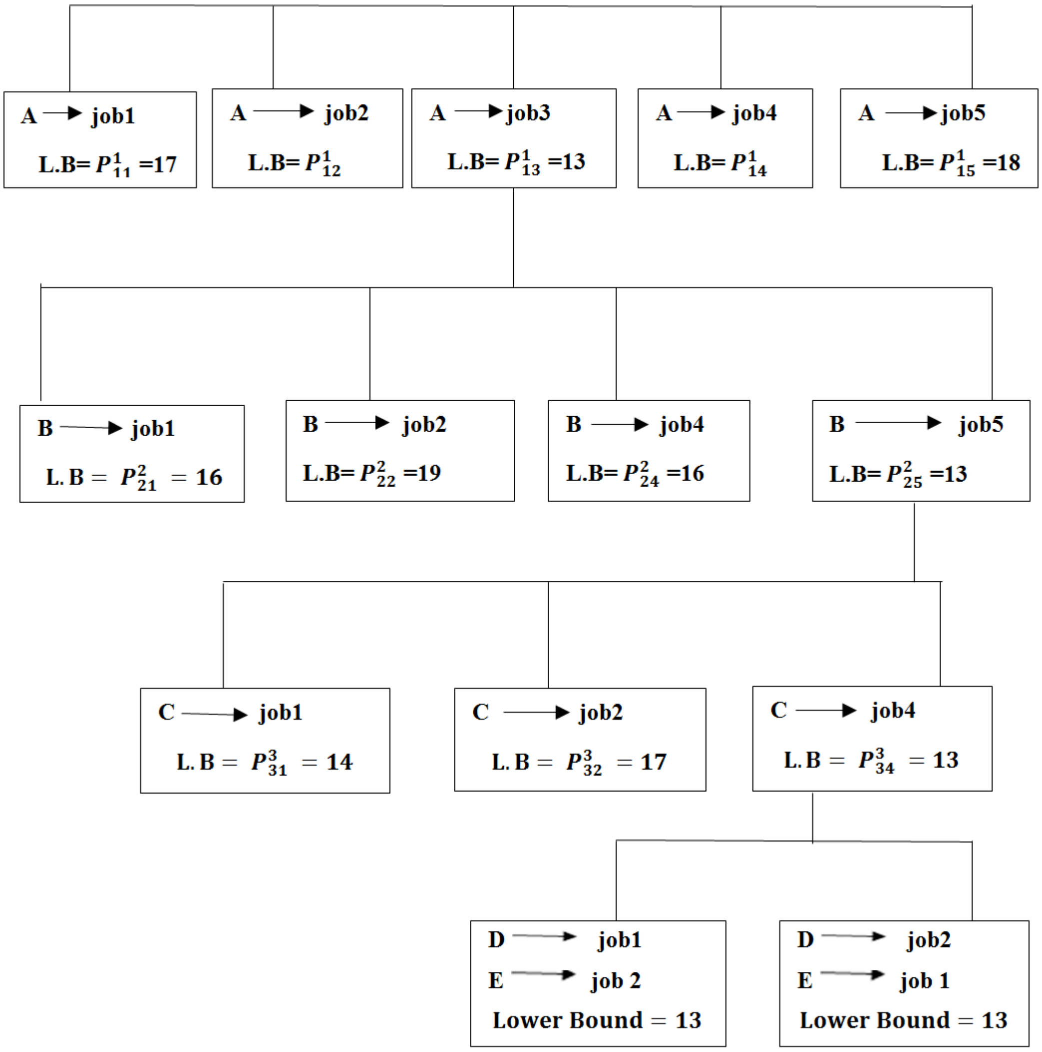

Figure 14. *Lower bound cost for assigning job to candidate D and E.*

Here both route have cost 13 same so we have two paths one is A to job3 B to job5 C to job4 D to job1 and E to job2. Another route is A to job3, B to job5, C to job4, D to job2 and E to job1. Minimum cost is 13. When we solve it, we receive the optimal objective value 13, which reflects the best total cost.

4. Conclusion

In this paper, a mathematical based model is suggested to assign the agents to the organization or firm where the established Hungarian method have not been used but we use both row and column operation. After both row and column operation with branch and bound technique followed by integer programming we get the same optimal value as found by the established Hungarian method.

The implementation of these agents allocation problem in firm was time consuming. Though the computational perception is limited, our algorithm appears to be fairly effective. Despite its simplicity, the proposed solution algorithm gives us the same solution obtained by the established Hungarian method.

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