



# On Computing the Metric Dimension of the Families of Alternate Snake Graphs

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**Abstract:** Consider a robot that is trying to determine its current location while navigating a graph-based environment. To know how distant it is from each group of fixed landmarks, it can send a signal. We handle the problem of precisely identifying the minimum number of landmarks needed and their ideal placement to guarantee the robot can always discover itself. The number of landmarks in the graph is its metric dimension, and the collection of nodes on which they are distributed is its metric basis. The smallest group of nodes required to uniquely identify each other node in a graph using shortest path distances is known as the metric dimension of the graph. We consider the NP-hard problem of finding the metric dimension of graphs. A set of vertices  $B$  of a connected graph  $G$  resolves  $G$  if every vertex of  $G$  is uniquely identified by its vector of distances to the vertices in  $B$ . The minimum resolving set is called the metric basis and the cardinality of the basis is called the metric dimension of  $G$ . Metric dimension has applications in a wide range of areas such as robot navigation, telecommunications, combinatorial optimization, and pharmacocatal chemistry. In this paper, we determine the metric dimension of the family of alternate snake graphs including alternate snake, alternate  $k$ -polygonal snake, double alternate triangular snake and triple alternate triangular snake graph.

**Keywords:** Metric Basis, Metric Dimension, Alternate Snake Graphs

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## 1. Introduction

Let  $G=(V, E)$  be a connected graph and  $d(u,v)$  be the shortest path between two vertices  $u,v \in V(G)$ . An ordered vertex set  $B=\{x_1, x_2, \dots, x_k\} \subseteq V(G)$  is a metric basis of  $G$  if the following two conditions are satisfied:

i)  $\forall v \in V(G)$ , the representation  $r(v|B) = (d(v, x_1), d(v, x_2), \dots, d(v, x_k))$  is unique.

ii)  $B$  with minimum cardinality.

The cardinality number of  $B$  is the metric dimension of  $G$  and is denoted  $\dim(G)$ .

Slater [1, 2] introduced the notion of a metric basis as a locating set of  $G$  and uses the cardinality of  $B$  as a locating number to uniquely identify the location of an intruder in a network. Harary and Melter [3] introduced independently the notion of metric basis as a resolving set of  $G$  and the cardinality

of  $B$  as a metric dimension. In [4], Khuller et al. have shown that determining the metric dimension of a graph is an NP-complete problem. Many scholars have improved an upper bound for the metric dimension of several graphs or determined their exact values. In [9], Saputro et al. obtained a sharp bound for the metric dimension of the lexicographic product of a connected graph  $G$  and an arbitrary graph  $H$ . In [6], Chartrand et al. showed that the path graph  $P_n$ ,  $n \geq 2$  has a constant metric dimension 1, cycle graph  $C_n$ ,  $n \geq 3$  has metric dimension 2, complete graph  $K_n$ ,  $n \geq 2$  has metric dimension  $n-1$  and complete bipartite graph  $K_{s,t}$ ,  $s, t \geq n \geq 4$  has metric dimension  $n-2$ .  $T$  is a tree of metric dimension 1, if  $T$  is a path.

In [10], Nawaz et al. proved that the total graph of path power three  $T(P_n^3)$  and four  $T(P_n^4)$  has an unbounded metric dimension. In [11], Nazeer et al. found that the metric dimension of two-middle path graph  $Two-Mid(p_q)$ ,  $q \geq 3$  is 2, three-middle path graph  $Three-Mid(P_q)$ ,  $q \geq 3$  is

3, three-total  $P_q$  Three- $T(P_q)$ ,  $q \geq 3$  is 3, reflection middle tower path graph  $RL(Tower_s)$ ,  $s \geq 3$  is 2, middle tower path graph  $Middle Tower_s$ ,  $s=2$  is 1,  $Middle Tower_s$ ,  $s \geq 3$  is 2, symmetrical planar pyramid graph  $SPPs$  is 2 and reflection symmetrical planar pyramid graph  $RL(SPPs)$  is 2. In [12], Ahmad et al. determined the metric dimension of kayak paddle graph  $KP(\ell, m, n)$  and cycles  $C_n$  with chord and proved that both families possess a metric dimension 2. In [13], Borchert et al. computed the metric dimension of the circulant graphs  $C_n(\pm 1, \pm 2)$  and have shown that if  $n \equiv 1 \pmod{4}$ , then  $\dim(C_n(\pm 1, \pm 2)) = 4$ . In [14], Imran et al. investigated the metric dimension of barycentric subdivision of Möbius ladders, the generalized Petersen multigraphs  $P(2n, n)$  and proved that they also have metric dimension 3 when  $n$  is even and 4 when  $n$  is odd. In [15], Jäger et al. proved that the metric dimension for  $Z_n \times Z_n \times Z_n$ ,  $n \geq 2$  is  $\lfloor \frac{3n}{2} \rfloor$ . In [16], Ahmad et al. proved that the metric dimension of  $P(n, 2) \circ K_1$  graph is 3, where  $\circ$  is corona product. The metric dimension of convex polytopes has been studied in [17-20]. In [21], Imran et al. proved that the metric dimension of the  $m$ -level gear graph  $J_{2n, m}$   $n \geq 4$ ,  $m \geq 3$  is  $\lfloor \frac{2n}{3} \rfloor + (m-1) \lfloor \frac{2n}{3} \rfloor$  and the metric dimension of the generalized gear graph  $J_{3n}$  is  $\lfloor \frac{n}{2} \rfloor$  for every  $n \geq 6$ . In [22], Pan et al. showed that the metric dimension of the splitting graphs of path  $S(P_n)$  and cycle  $S(C_n)$ ,  $n > 8$  are  $\lfloor \frac{n}{3} \rfloor$ . In [23], Siddiqui et al. computed the metric dimension of antiweb-gear graphs  $AWJ_{2n}$ ,  $n \geq 15$  is  $\lfloor \frac{n+1}{3} \rfloor$ ,  $m$ -level wheel  $W_{n, m}$ ,  $n \geq 7$ ,  $m \geq 3$  is  $\lfloor \frac{2n+2}{5} \rfloor + (m-1) \lfloor \frac{2n+4}{5} \rfloor$ . The first attempt to heuristically compute the smallest connected dominant resolving set of graphs using a binary version of the equilibrium optimization algorithm was made by Mohamed et al. [24]. The first binary implementation of the Enhanced Harris Hawks Optimization was provided by Mohamed et al. [25] in an

attempt to heuristically compute the minimal connected resolving set of graphs. Mohamed et al. [26] investigated the metric dimension of subdivisions of a number of graphs, including the Lilly graph, the Tadpole graph, and the special trees star tree, bistar tree, and coconut tree. Mohamed et al. [27] examined the specific value of the secure resolving set for a few networks, including the trapezoid network,  $Z(P_n)$  network, open ladder network, tortoise network, and  $P_{2n} \bar{\vee} P_n$  network. Additionally, they calculated the domination numbers for several networks, including the twig network, double fan network, bistar network and linear  $kc_4$  - snake network. Mohamed [28] focused on the contraction and bijection of the metric dimension when a robot is moving across a network that is modelled by the  $(2, 1)$   $C_4$ -snake graph,  $2\Delta_2$ -snake graph and  $3C_4$ -snake graph.

Metric dimension has been used in several applications such as robot navigation in networks [4, 5, 29-31], application to pharmaceutical chemistry Chartrand et al. [[6]], application to pattern recognition Melter et al. [7], and application to wireless sensor network localization [8].

In this paper, the metric dimension of alternate snake graphs, double alternate triangular snake, and alternate triple triangular snake graph is investigated.

## 2. The Metric Dimension for Alternate Snake Graphs

*Theorem 2.1*  $\dim(G) = 2$  if  $G$  is

- the alternate triangular snake  $A(T_n)$ ,  $n > 3$ .
- the alternate quadrilateral snake  $A(QS_n)$ ,  $n \geq 8$ .
- the alternate pentagonal snake  $A(PS_n)$ ,  $n \geq 5$ .

*Proof.* i) Consider the following cases of alternate triangular snake  $A(T_n)$ .

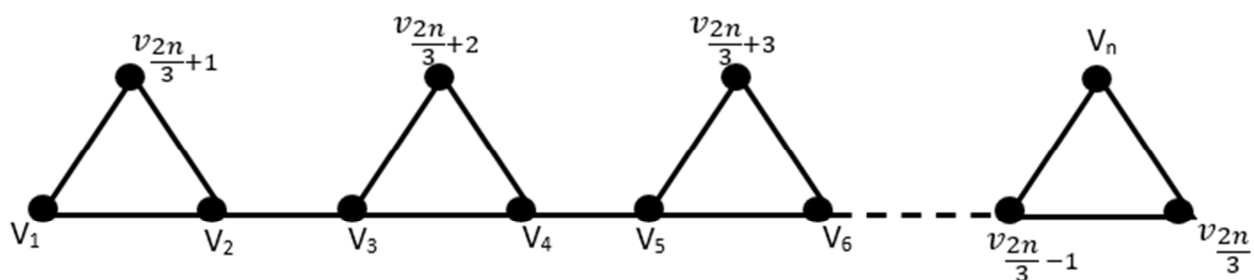


Figure 1. Alternate triangular snake  $A(T_n)$ .

*Case 1.*  $n \equiv 0 \pmod{3}$

Let  $n=3k$ ,  $k > 1$ ,  $k \in \mathbb{Z}^+$ . The resolving set in general form is  $B = \{v_1, v_{2k}\} \subset V(A(T_n))$  as well as the representations of vertices  $v_i \in V(A(T_n))$  in regard to  $B$  are

$$r(v_i|B) = \begin{cases} (i-1, 2k-i), & 1 \leq i \leq 2k; \\ (2i-n-5, 2n-2i+1), & 2k+1 \leq i \leq n. \end{cases}$$

Since all vertices have unique representations, we obtain

$\dim(A(T_n)) = 2$ .

$B$  is also minimal, since  $Q = \{v_1, v_2\} \subset B$  and  $T = \{v_1, v_3\} \subset B$  are not resolving sets:

Vertices  $v_{n-k}$  and  $v_n$  have equal distances to vertices of  $Q$  and  $T$ .

*Case 2.*  $n \equiv 1 \pmod{3}$

Let  $n=3k+1$ ,  $k > 1$ ,  $k \in \mathbb{Z}^+$ . The resolving set in general form is  $B = \{v_1, v_{2k}\} \subset V(A(T_n))$  as well as the representations of vertices  $v_i \in V(A(T_n))$  in regard to  $B$  are

$$r(v_i|B) = \begin{cases} (i-1, 2k-i), & 1 \leq i \leq 2k; \\ (i-1, 1) & i = 2k+1 \\ (2i-n-5, 2n-2i+1), & 2k+2 \leq i \leq n. \end{cases}$$

Since all vertices have unique representations, we obtain  $\dim(A(T_n)) = 2$ .

$B$  is also minimal, since  $Q = \{v_1, v_2\} \subset B$  and  $T = \{v_1, v_3\} \subset B$  are not resolving sets: vertices  $v_{n-k-1}$  and  $v_n$  have equal distances to vertices of  $Q$  and  $T$ .

Case 3.  $n \equiv 2(\text{mod } 3)$

Let  $n = 3k+2$ ,  $k > 1$ ,  $k \in \mathbb{Z}^+$ . The resolving set is  $B = \{v_1, v_{2k+1}\} \subset V(A(T_n))$  as well as the representations of vertices  $v_i \in V(A(T_n))$  with regard to  $B$  are

$$r(v_i|B) = \begin{cases} (i-1, 2k-i+1), & 1 \leq i \leq 2k+1; \\ (i-1, 1), & i = 2k+2; \\ (2i-n-5, 2n-2i+1), & 2k+3 \leq i \leq n. \end{cases}$$

Since all vertices have unique representations, we obtain  $\dim(A(T_n)) = 2$ .

$B$  is also minimal, since  $Q = \{v_1, v_2\} \subset B$  is not resolving set: vertices  $v_3$  and  $v_{n-k+1}$  have equal distance to vertices of  $Q$ .

ii) The alternate quadrilateral snake  $A(QS_n)$  has the following cases:

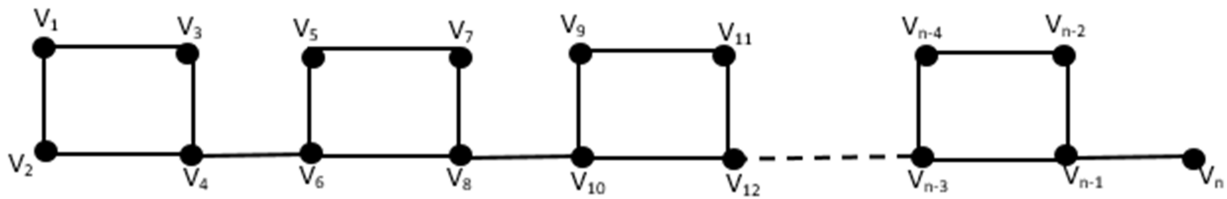


Figure 2. Alternate quadrilateral snake  $A(QS_n)$ .

Case 1:  $n \equiv 0(\text{mod } 4)$

Let  $n = 4k$ ,  $k \geq 2$ ,  $k \in \mathbb{Z}^+$ . The resolving set in general form is  $B = \{v_2, v_{n-3}\} \subset V(A(QS_n))$  as well as the representations of vertices  $v_i \in V(A(QS_n))$  in regard to  $B$  are as follow:

$$r(v_i|B) = \begin{cases} \left(\frac{i+1}{2}, \frac{n-i+1}{2}\right), & 1 \leq i \leq n-5; \text{ for } i=1,3,\dots \\ \left(\frac{i-2}{2}, \frac{n-i}{2}\right), & 2 \leq i \leq n-4; \text{ for } i=2,4,\dots \\ \left(\frac{i+1}{2}, 0\right), & i = n-3; \\ (2k-2, 1), & i = n-2; \\ \left(\frac{i+1}{2}, 1\right), & i = n-1. \\ (2k-1, 2), & i = n. \end{cases}$$

These representations are concluded to be distinct and  $\dim(A(QS_n)) = 2$ .

$B$  is also minimal, since  $Q = \{v_2, v_3\} \subset B$  and  $T = \{v_2, v_4\} \subset B$  are not resolving sets: vertices  $v_1$  and  $v_4$  have equal distance to vertices of  $Q$  and vertices  $v_1$  and  $v_3$  of  $T$ .

Case 2:  $n \equiv 1(\text{mod } 4)$

Let  $n = 4k+1$ ,  $k \geq 2$ ,  $k \in \mathbb{Z}^+$ . The resolving set in general form is  $B = \{v_2, v_{n-4}\} \subset V(A(QS_n))$  as well as the representations of vertices  $v_i \in V(A(QS_n))$  in regard to  $B$  are as follow:

$$r(v_i|B) = \begin{cases} \left(\frac{i+1}{2}, \frac{n-i}{2}\right), & 1 \leq i \leq n-6; \text{ for } i=1,3,\dots \\ \left(\frac{i-2}{2}, \frac{n-i-1}{2}\right), & 2 \leq i \leq n-5; \text{ for } i=2,4,\dots \\ (2k-1, 0), & i = n-4; \\ \left(\frac{i-2}{2}, 1\right), & i = n-3; \\ (2k, 1), & i = n-2; \\ \left(\frac{i-2}{2}, 2\right), & i = n-1. \\ (2k, 3), & i = n. \end{cases}$$

These representations are concluded to be distinct and  $\dim(A(QS_n)) = 2$ .

$B$  is also minimal, since  $Q = \{v_2, v_{n-5}\} \subset B$  and  $T = \{v_2, v_{n-6}\} \subset B$  are not resolving sets:

vertices  $v_{n-1}$  and  $v_{n-4}$  have equal distance to vertices of  $Q$  and  $T$ .

Case 3:  $n \equiv 2(\text{mod } 4)$

Let  $n = 4k+2$ ,  $k \geq 2$ ,  $k \in \mathbb{Z}^+$ . The resolving set in general form is  $B = \{v_1, v_{n-4}\} \subset V(A(QS_n))$  as well as the representations of vertices  $v_i \in V(A(QS_n))$  in regard to  $B$  are as follow:

$$r(v_i|B) = \begin{cases} \left(\frac{i-1}{2}, \frac{n-i-1}{2}\right), & 1 \leq i \leq n-3; \text{ for } i=1,3,\dots \\ \left(\frac{i+2}{2}, \frac{n-i}{2}\right), & 2 \leq i \leq n-2; \text{ for } i=2,4,\dots \\ \left(\frac{i-1}{2}, 2\right), & i = n-1. \\ \left(\frac{i}{2}, 3\right), & i = n. \end{cases}$$

These representations are concluded to be distinct and  $\dim(A(QS_n)) = 2$ .

$B$  is also minimal, since  $Q = \{v_1, v_{n-5}\} \subset B$  and  $T = \{v_1, v_{n-7}\} \subset B$  are not resolving sets: vertices  $v_{n-1}$  and  $v_{n-4}$  have equal distance to vertices of  $Q$  and  $T$ .

iii) The alternate pentagonal snake  $A(PS_n)$ .

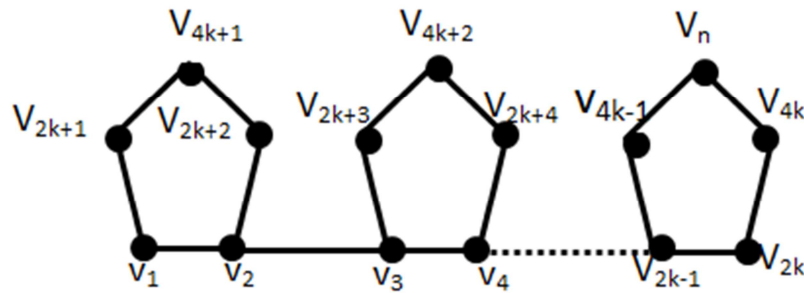


Figure 3. Alternate pentagonal snake  $A(PS_n)$ .

Case 1.  $n \equiv 0 \pmod{5}$

Let  $n = 5k$ ,  $k \geq 1$ ,  $k \in \mathbb{Z}^+$ . A minimal resolving set of  $V(A(PS_n))$  is  $B = \{v_1, v_{2k}\} \subset V(A(PS_n))$ . The following representations of vertices  $v_i \in V(A(PS_n))$  in regard to  $B$  are distinct.

$$r(v_i|B) = \begin{cases} (i-1, 2k-i), & 1 \leq i \leq 2k; \\ (i-2k, n-i-2), & 2k+1 \leq i \leq 4k; \\ (2i-9k+3, 2n-2i+2), & 4k+1 \leq i \leq n. \end{cases}$$

Thus,  $\dim(A(PS_n)) = 2$ .

Case 2.  $n \equiv 1 \pmod{5}$

Let  $n = 5k+1$ ,  $k \geq 1$ ,  $k \in \mathbb{Z}^+$ . A minimal resolving set of  $V(A(PS_n))$  is  $B = \{v_1, v_{2k+1}\} \subset V(A(PS_n))$ . The following representations of vertices  $v_i \in V(A(PS_n))$  in regard to  $B$  are distinct.

$$r(v_i|B) = \begin{cases} (i-1, 2k-i+1), & 1 \leq i \leq 2k+1; \\ (i-2k-1, n-i-1), & 2k+2 \leq i \leq 4k+1; \\ (2i-9k+1, 2n-2i+3), & 4k+2 \leq i \leq n. \end{cases}$$

Thus,  $\dim(A(PS_n)) = 2$ .

Case 3.  $n \equiv 2 \pmod{5}$

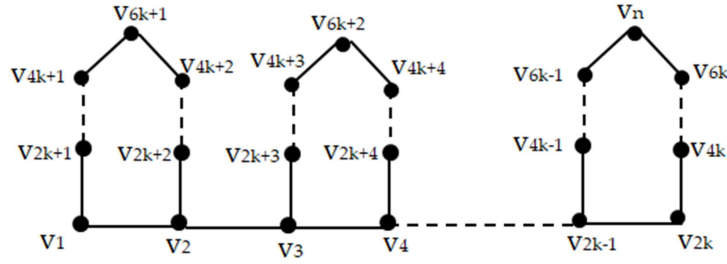
Let  $n = 5k+2$ ,  $k \geq 1$ ,  $k \in \mathbb{Z}^+$ . The resolving set in general form is  $B = \{v_1, v_{2k+2}\} \subset V(A(PPS_n))$ . The representations of vertices  $v_i \in V(A(PPS_n))$  with regard to  $B$  are

$$r(v_i|B) = \begin{cases} (i-1, 2k+2-i), & 1 \leq i \leq 2k+2; \\ (i-2k-1, n-i), & 2k+3 \leq i \leq 4k+2; \\ (2i-9k-1, 2n-2i+3), & 4k+3 \leq i \leq n. \end{cases}$$

Since all vertices have different representations, we obtain  $\dim(A(PPS_n)) = 2$ .

From case (1), case (2) and case (3), We conclude that metric dimension of alternate snake is 2.

**Theorem 2.2.** If  $AS_n(C_k)$ ,  $n \geq 3$ ,  $k \geq 1$  is an alternate  $k$ -polygonal snake, then  $\dim(AS_n(C_k))=2$ .



**Figure 4.** Alternate  $k$ -pentagonal snake  $AS_n(C_k)$ .

*Proof.* Consider the following cases of alternate  $k$ -pentagonal snake  $AS_n(C_k)$ .

Case 1.  $n \equiv 0(\text{mod } k)$

A minimal resolving set of  $V(AS_n(C_k))$  is  $B = \{v_1, v_{2k}\} \subset V(AS_n(C_k))$ . The following representations of vertices  $v_i \in V(AS_n(C_k))$  in regard to  $B$  are distinct.

$$r(v_i|B) = \begin{cases} (i-1, 2k-i), & 1 \leq i \leq 2k; \\ (i-2k, 4k-i+1), & 2k+1 \leq i \leq 4k; \\ (i-4k+1, 6k-i+2), & 4k+1 \leq i \leq 6k; \\ (2i-12k+1, 2n-2i+3), & 6k+1 \leq i \leq n. \end{cases}$$

Thus,  $\dim(AS_n(C_k)) = 2$ .

Case 2.  $n \equiv 1(\text{mod } k)$

A minimal resolving set of  $V(AS_n(C_k))$  is  $B = \{v_1, v_{2k+1}\} \subset V(AS_n(C_k))$ . The following representations of vertices  $v_i \in V(AS_n(C_k))$  in regard to  $B$  are distinct.

$$r(v_i|B) = \begin{cases} (i-1, 2k-i+1), & 1 \leq i \leq 2k+1; \\ (i-2k-1, 4k-i+3), & 2k+2 \leq i \leq 4k+1; \\ (i-4k, 6k-i+4), & 4k+2 \leq i \leq 6k+1; \\ (2i-12k-1, 2n-2i+4), & 6k+2 \leq i \leq n. \end{cases}$$

Thus,  $\dim(AS_n(C_k)) = 2$ .

Case 3.  $n \equiv 2(\text{mod } k)$

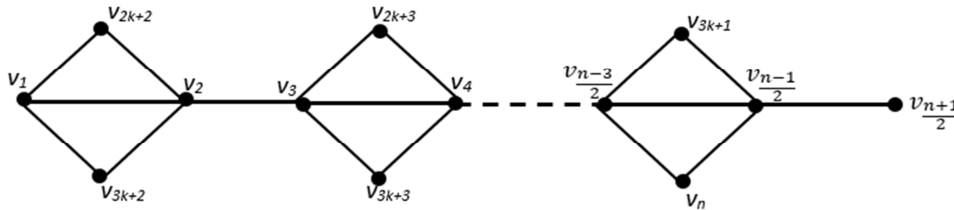
A minimal resolving set of  $V(AS_n(C_k))$  is  $B = \{v_1, v_{2k+2}\} \subset V(AS_n(C_k))$ . The following representations of vertices  $v_i \in V(AS_n(C_k))$  in regard to  $B$  are distinct.

$$r(v_i|B) = \begin{cases} (i-1, 2k-i+2), & 1 \leq i \leq 2k+4; \\ (i-2k-1, 4k-i+4), & 2k+5 \leq i \leq 4k+2; \\ (i-4k-1, 6k-i+6), & 4k+3 \leq i \leq 6k+2; \\ (2i-12k+5, 2n-2i+3), & 6k+3 \leq i \leq n. \end{cases}$$

Thus,  $\dim(AS_n(C_k)) = 2$ .

**Theorem 2.3.** If  $DA(T_n)$ ,  $n \geq 8$  is a double alternate triangular snake, then

$$\dim(DA(T_n)) = \begin{cases} \frac{n}{4} & \text{if } n = 8, 12, \dots \\ \frac{n-1}{4} & \text{if } n = 9, 13, \dots \\ \frac{n-2}{4} & \text{if } n = 10, 14, \dots \end{cases}$$



**Figure 5.** Double alternate triangular snake  $DA(T_n)$ .

*Proof.* The double alternate triangular snake has two cases.

Case 1. When  $k$  is even ( $k$  is the number of blocks)

Subcase 1.  $n \equiv 0(\text{mod } 4)$

We write  $n=4k$ ,  $k \geq 2$ ,  $k \in \mathbb{Z}^+$ . The resolving set in general form is  $B = \{v_{2k+1}, v_{2k+2}, v_{3k}\} \subset V(DA(T_n))$  as well as the representations of vertices  $v_i \in V(DA(T_n))$  in regard to  $B$  are

$$r(v_i|B) = \left\{ \begin{array}{ll} (1,3,5, \dots, 2k-1), & i=1; \\ (1,2,4, \dots, 2k-2), & i=2; \\ (2,1,3, \dots, 2k-3), & i=3; \\ (3,1,2, \dots, 2k-4), & i=4; \\ (i-1, i-3, \dots, 1,3,5, \dots, 2k-i), & 5 \leq i \leq 2k-1, \text{ where } i \text{ is odd;} \\ (i-1, i-3, \dots, 1,2,4, \dots, 2k-i), & 6 \leq i \leq 2k-2, \text{ where } i \text{ is even;} \\ (i-1, i-3, \dots, 1), & i=2k; \\ (0,3,5, \dots, 2k-1), & i=2k+1; \\ (3,0,3, \dots, 2k-3), & i=2k+2; \\ (2i-5k+5, 2i-5k+3, \dots, 0,3,5, \dots, 7k-2i-1), & 2k+3 \leq i \leq 3k-1; \\ (2i-5k+5, 2i-5k+3, \dots, 0), & i=3k; \\ (2,3,5, \dots, 2k-1), & i=3k+1; \\ (3,2,3, \dots, 2k-3), & i=3k+2; \\ (2i-7k+5, 2i-7k+3, \dots, 2,3,5, \dots, 9k-2i-5), & 3k+3 \leq i \leq n-1; \\ (2k-1, 2k-3, \dots, 3,2), & i=n. \end{array} \right.$$

Assume that  $B = \{v_{2k+1}, v_{2k+2}, \dots, v_{3k}\}$  is the basis for  $DA(T_n)$ . Every element in  $V(DA(T_n))$  has unique representation. So  $B$  is resolving set for  $DA(T_n)$ .

If we remove any vertex from  $B$  then it is not resolving set for  $DA(T_n)$ . Let  $B' = \{v_{2k+1}, v_{2k+2}, \dots, v_{3k-i}\}$  be subset of  $V(DA(T_n))$  then there is at least one pair of vertices which gives same representation. So  $B'$  is not resolving set for  $DA(T_n)$ . Hence

$$\dim(DA(T_n)) = \frac{n}{4}.$$

Subcase 2.  $n \equiv 1 \pmod{4}$

Let  $n=4k+1$ ,  $k \geq 2$ ,  $k \in \mathbb{Z}^+$ . The resolving set in general form is  $B = \{v_{2k+2}, v_{2k+3}, \dots, v_{3k+1}\} \subseteq V(DA(T_n))$  as well as the representations of vertices  $v_i \in V(DA(T_n))$  in regard to  $B$  are

$$r(v_i|B) = \left\{ \begin{array}{ll} (1,3,5, \dots, 2k-1), & i=1; \\ (1,2,4, \dots, 2k-2), & i=2; \\ (i-1, i-3, \dots, 1,3,5, \dots, 2k-i), & 3 \leq i \leq 2k-1, \text{ where } i \text{ is odd;} \\ (i-1, i-3, \dots, 1,2,4, \dots, 2k-i), & 4 \leq i \leq 2k-2, \text{ where } i \text{ is even;} \\ (i-1, i-3, \dots, 1), & i=2k; \\ (i-1, i-3, \dots, 2), & i=2k+1; \\ (0,3,5, \dots, 2k-1), & i=2k+2; \\ (2i-5k+5, 2i-5k+3, \dots, 0,3,5, \dots, 7k-2i-5), & 2k+3 \leq i \leq 3k; \\ (i-k-2, i-k-4, \dots, 0), & i=3k+1; \\ (2,3,5, \dots, 2k-1), & i=3k+2; \\ (2i-7k+5, 2i-7k+3, \dots, 2,3,5, \dots, 9k-2i-5), & 3k+3 \leq i \leq n-1; \\ (2i-7k+5, 2i-7k+3, \dots, 2), & i=n. \end{array} \right.$$

Since all vertices have different representations, we obtain  $\dim(DA(T_n)) = \frac{n-1}{4}$ .

*Subcase 3.  $n \equiv 2 \pmod{4}$*

Let  $n=4k+2$ ,  $k \geq 2, k \in \mathbb{Z}^+$ . The resolving set in general form is  $B = \{v_{2k+3}, v_{2k+4}, \dots, v_{3k+2}\} \cup V(DA(T_n))$  as well as the representations of vertices  $v_i \in V(DA(T_n))$  in regard to  $B$  are

$$r(v_i|B) = \left\{ \begin{array}{ll} (2, 4, 6, \dots, 2k), & i = 1; \\ (1, 3, 5, \dots, 2k-1), & i = 2; \\ (1, 2, 4, \dots, 2k-2), & i = 3; \\ (2, 1, 3, \dots, 2k-3), & i = 4; \\ (i-2, i-4, \dots, 1, 2, \dots, 2k-i+1), & 5 \leq i \leq 2k-1, \text{ where } i \text{ is odd;} \\ (i-2, i-4, \dots, 2, 1, \dots, 2k-i+1), & 6 \leq i \leq 2k, \text{ where } i \text{ is even;} \\ (i-2, i-4, \dots, \dots, 3, 1), & i=2k+1; \\ (i-2, i-4, \dots, \dots, 4, 2), & i=2k+2; \\ (0, 3, 5, \dots, 2k-1), & i=2k+3; \\ (2i-5k+1, 2i-5k-1, \dots, 0, 3, 5, \dots, 7k-2i-1), & 2k+4 \leq i \leq 3k+1; \\ (i-k-3, i-k-5, \dots, 0), & i=3k+2; \\ (2i-7k+1, 2i-7k-1, \dots, 2, 3, 5, \dots, 9k-2i-1), & 3k+3 \leq i \leq n-1; \\ (2k-1, 2k-3, \dots, 3, 2), & i=n. \end{array} \right.$$

Since all vertices have different representations, we obtain  $\dim(DA(T_n)) = \frac{n-2}{4}$ .

*Case 2. When  $k$  is odd*

*Subcase 1.  $n \equiv 0 \pmod{4}$*

Let  $n=4k$ ,  $k \geq 3, k \in \mathbb{Z}^+$ . The resolving set in general form is  $B = \{v_{2k+1}, v_{2k+2}, \dots, v_{3k}\} \cup V(DA(T_n))$  as well as the representations of vertices  $v_i \in V(DA(T_n))$  in regard to  $B$  are

$$r(v_i|B) = \left\{ \begin{array}{ll} (1, 3, 5, \dots, 2k-1), & i = 1; \\ (1, 2, 4, \dots, 2k-2), & i = 2; \\ (i-1, i-3, \dots, 1, 3, 5, \dots, 2k-i), & 3 \leq i \leq 2k-1, \text{ where } i \text{ is odd;} \\ (i-1, i-3, \dots, 1, 2, 4, \dots, 2k-i), & 4 \leq i \leq 2k-2, \text{ where } i \text{ is even;} \\ (i-1, i-3, \dots, 1), & i=2k; \\ (0, 3, 5, \dots, 2k-1), & i=2k+1; \\ (2i-5k+4, 2i-5k+2, \dots, 0, 3, 5, \dots, 7k-2i-4), & 2k+2 \leq i \leq 3k-1; \\ (i-k-1, i-k-3, \dots, 0), & i=3k; \\ (2, 3, 5, \dots, 2k-1), & i=3k+1; \\ (2i-6k-1, 2i-6k-3, \dots, 2, 3, 5, \dots, 8k-2i+1), & 3k+2 \leq i \leq n-1; \\ (2k-1, 2k-3, \dots, 3, 2), & i=n. \end{array} \right.$$

We assume that  $B = \{v_{2k+1}, v_{2k+2}, \dots, v_{3k}\}$  is the basis for  $DA(T_n)$ . Every element in  $V(DA(T_n))$  has unique representation. So  $B$  is resolving set for  $DA(T_n)$ .

If we remove any vertex from  $B$  then it is not resolving set for  $DA(T_n)$ . Let  $B' = \{v_{2k+1}, v_{2k+2}, \dots, v_{3k-1}\}$  be subset of  $V(DA(T_n))$  then there is at least one pair of vertices which gives same representation. So  $B'$  is not resolving set for  $DA(T_n)$ . Hence  $\dim(DA(T_n)) = \frac{n-2}{4}$ .

*Subcase 2.  $n \equiv 1 \pmod{4}$*

Let  $n=4k+1$ ,  $k \geq 3, k \in \mathbb{Z}^+$ . The resolving set in general form is  $B = \{v_{2k+3}, v_{2k+4}, \dots, v_{3k+2}\} \subseteq V(DA(T_n))$  as well as the representations of vertices  $v_i \in V(DA(T_n))$  in regard to  $B$  are

$$r(v_i|B) = \left\{ \begin{array}{l} (1, 3, 5, \dots, 2k-1), \quad i=1; \\ (1, 2, 4, \dots, 2k-2), \quad i=2; \\ (i-1, i-3, \dots, 1, 3, 5, \dots, 2k-i), \quad 3 \leq i \leq 2k-1, \text{ where } i \text{ is odd;} \\ (i-1, i-3, \dots, 1, 2, 4, \dots, 2k-i), \quad 4 \leq i \leq 2k-2, \text{ where } i \text{ is even;} \\ (i-1, i-3, \dots, 1), \quad i=2k; \\ (i-1, i-3, \dots, 2), \quad i=2k+1; \\ (0, 3, 5, \dots, 2k-1), \quad i=2k+2; \\ (2i-5k+2, 2i-5k, \dots, 0, 3, 5, \dots, 7k-2i-2), \quad 2k+3 \leq i \leq 3k; \\ (i-k-2, i-k-4, \dots, 0), \quad i=3k+1; \\ (2, 3, 5, \dots, 2k-1), \quad i=3k+2; \\ (2i-7k+2, 2i-7k, \dots, 2, 3, 5, \dots, 9k-2i-2), \quad 3k+3 \leq i \leq n-1; \\ (2i-7k+2, 2i-7k, \dots, 2), \quad i=n. \end{array} \right.$$

As all representations are distinct, we conclude that  $\dim(DA(T_n)) = \frac{n-1}{4}$ .

*Subcase 3.  $n \equiv 2 \pmod{4}$*

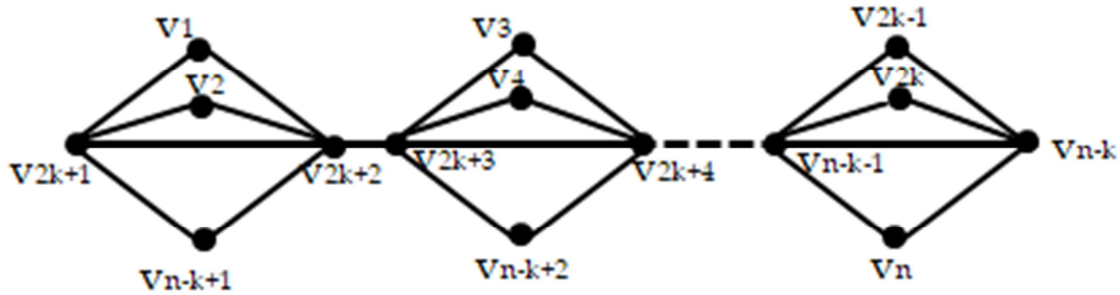
Let  $n=4k+2$ ,  $k \geq 3, k \in \mathbb{Z}^+$ . The resolving set in general form is  $B = \{v_{2k+3}, v_{2k+4}, \dots, v_{3k+2}\} \subseteq V(DA(T_n))$  as well as the representations of vertices  $v_i \in V(DA(T_n))$  in regard to  $B$  are

$$r(v_i|B) = \left\{ \begin{array}{l} (2, 4, \dots, 2k), \quad i=1; \\ (1, 3, \dots, 2k-1), \quad i=2; \\ (i-2, i-4, \dots, 1, 2, 4, \dots, 2k-i+1), \quad 3 \leq i \leq 2k-1, \text{ where } i \text{ is odd;} \\ (i-2, i-4, \dots, 2, 1, 3, \dots, 2k-i+1), \quad 4 \leq i \leq 2k-2, \text{ where } i \text{ is even;} \\ (i-2, i-4, \dots, 1), \quad i=2k, 2k+1; \\ (i-2, i-4, \dots, 2), \quad i=2k+2; \\ (0, 3, 5, \dots, 2k-1), \quad i=2k+3; \\ (2i-5k+2, 2i-5k, \dots, 0, 3, 5, \dots, 7k-2i), \quad 2k+4 \leq i \leq 3k+2; \\ (2, 3, 5, \dots, 2k-1), \quad i=3k+3; \\ (2i-7k, 2i-7k-2, \dots, 2, 3, 5, \dots, 9k-2i), \quad 3k+4 \leq i \leq n-1; \\ (2k-1, 2k-3, \dots, 3, 2), \quad i=n. \end{array} \right.$$



As all representations are distinct, we conclude that  $\dim(DA(T_n)) = \frac{n-2}{4}$ .

**Theorem 2.4.** If  $AT(T_n)$ ,  $n \geq 10$  is alternate triple triangular snake graph, then  $\dim(AT(T_n)) = \frac{n-k}{2}$ .



**Figure 6.** Alternate triple triangular Snake graph  $AT(T_n)$ .

*Proof.* Let  $n=5k$ ,  $k \geq 2, k \in \mathbb{Z}^+$ . The resolving set in general form is  $B = \{v_1, v_2, \dots, v_{2k}\} \subset V(AT(T_n))$  as well as the representations of vertices  $v_i \in V(AT(T_n))$  in regard to  $B$  are

$$r(v_i|B) = \left\{ \begin{array}{ll} (0, 2, 3, 3, 5, 5, \dots, 2k-1, 2k-1), & i=1; \\ (2, 0, 3, 3, 5, 5, \dots, 2k-1, 2k-1), & i=2; \\ (i, i, i-2, i-2, \dots, 3, 3, 0, 2, \dots, 2k-i, 2k-i), & 3 \leq i \leq 2k, \text{ where } i \text{ is odd}; \\ (i-1, i-1, i-3, i-3, \dots, 2, 0, 3, 3, \dots, 2k+1-i, 2k+1-i), & 4 \leq i \leq 2k, \text{ where } i \text{ is even}; \\ (i-1, i-1, i-3, i-3, \dots, 2, 0), & i=2k; \\ (1, 1, 3, 3, \dots, 2k-1, 2k-1), & i=2k+1; \\ (i-2k-1, i-2k-1, i-2k-3, i-2k-3, \dots, 1, 1, 2, 2, \dots, 4k-i, 4k-i), & 2k+2 \leq i \leq 4k-1; \\ (i-2k-1, i-2k-1, i-2k-3, i-2k-3, \dots, 1, 1), & i=4k; \\ (2, 2, 3, 3, 5, 5, \dots, 2k-1, 2k-1), & i=4k+1; \\ (3, 3, 2, 2, 3, 3, \dots, 2k-3, 2k-3), & i=4k+2; \\ (2i-8k-1, 2i-8k-1, 2i-8k-3, 2i-8k-3, \dots, 3, 3, 2, 2, \dots, 10k-2i+1, 10k-2i+1), & i=4k+3 \leq i \leq n-1; \\ (2k-1, 2k-1, 2k-3, 2k-3, \dots, 2, 2), & i=n. \end{array} \right.$$

We conclude that these representations are distinct and  $\dim(AT(T_n)) = \frac{n-k}{2}$ .

### 3. Conclusion

The metric dimension of alternate snake graphs have a constant metric dimension 2. The double alternate triangular snake and alternate triple triangular snake graph have an unbounded metric dimension as  $n \rightarrow \infty$ .

*Open problem:* Compute metric dimension of subdivision of alternate  $k$ -polygonal snake.

### Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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